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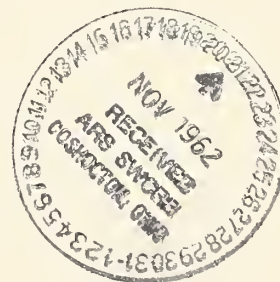
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# DETERMINATION OF SEDIMENT DENSITY BY GAMMA ATTENUATION



Growth Through Agricultural Progress



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## ABSTRACT

The measurement of density as a function of the attenuation of a radioactive emission is evaluated as a means of determining sediment density and concentration in silt boxes, laboratory flumes, and flowing streams. The transmission method, that is, the measurement of the attenuated primary emission, shows considerable promise on the basis of laboratory investigations. Plans for field installations are discussed.

# DETERMINATION OF SEDIMENT DENSITY BY GAMMA ATTENUATION<sup>1</sup>

J. Roger McHenry<sup>2</sup>

## INTRODUCTION

The need for precise measurements of the density of materials has increased in recent years. Formerly, density variations in materials were detected mainly by radiographic techniques utilizing high energy x-ray equipment. The requirements of greater precision, however, and the technical difficulties of using radiographic methods in many operations have led to the development of radiation gages employing radioactive material. The term DXT (density times thickness) has become associated with this type of measurement because, inherently, the device is unable to distinguish between changes in thickness and changes in density. A number of density gages have been developed for specific jobs (6, 7).<sup>3</sup> The possibility of devising such a system for evaluating the sediment density or concentration in silt boxes, laboratory flumes, and flowing streams has been investigated. The results of these investigations and proposals for future research work are presented in this report.

## THEORY

Gamma rays and x-rays are two forms of electromagnetic radiation differing only in their origin. Gamma rays are produced in nuclear reactions, whereas x-rays are caused by excitation of orbital electrons. The emission of gamma rays is a mechanism by which the excess energy of excitation of a nucleus can be removed (9). Such excited states may accompany the decay of radioisotopes, or they may result from induced nuclear transmutations. The gamma rays accompanying a particular type of nuclear reaction are composed of photons with either a single energy or a group of discrete energies. Typical energies of gamma rays range from a few Kev. to several Mev. The mass absorption coefficients are functions of both the energy of the photons and the atomic number of the absorber (Figure 1).

The absorption of either gamma or x-rays may be studied by measuring their transmission through absorbers. Consider an arrangement similar to that in Figure 2. The measure of the intensity of the beam that reaches the detector may be expressed:

$$\frac{dI}{I} = -\mu_L dx \quad (1)$$

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<sup>1</sup>Contribution from the USDA Sedimentation Laboratory, Soil and Water Conservation Research Division, Agricultural Research Service, United States Department of Agriculture, in cooperation with the University of Mississippi and Mississippi State University.

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<sup>3</sup>Figures in parentheses refer to Literature Cited at end of this publication.

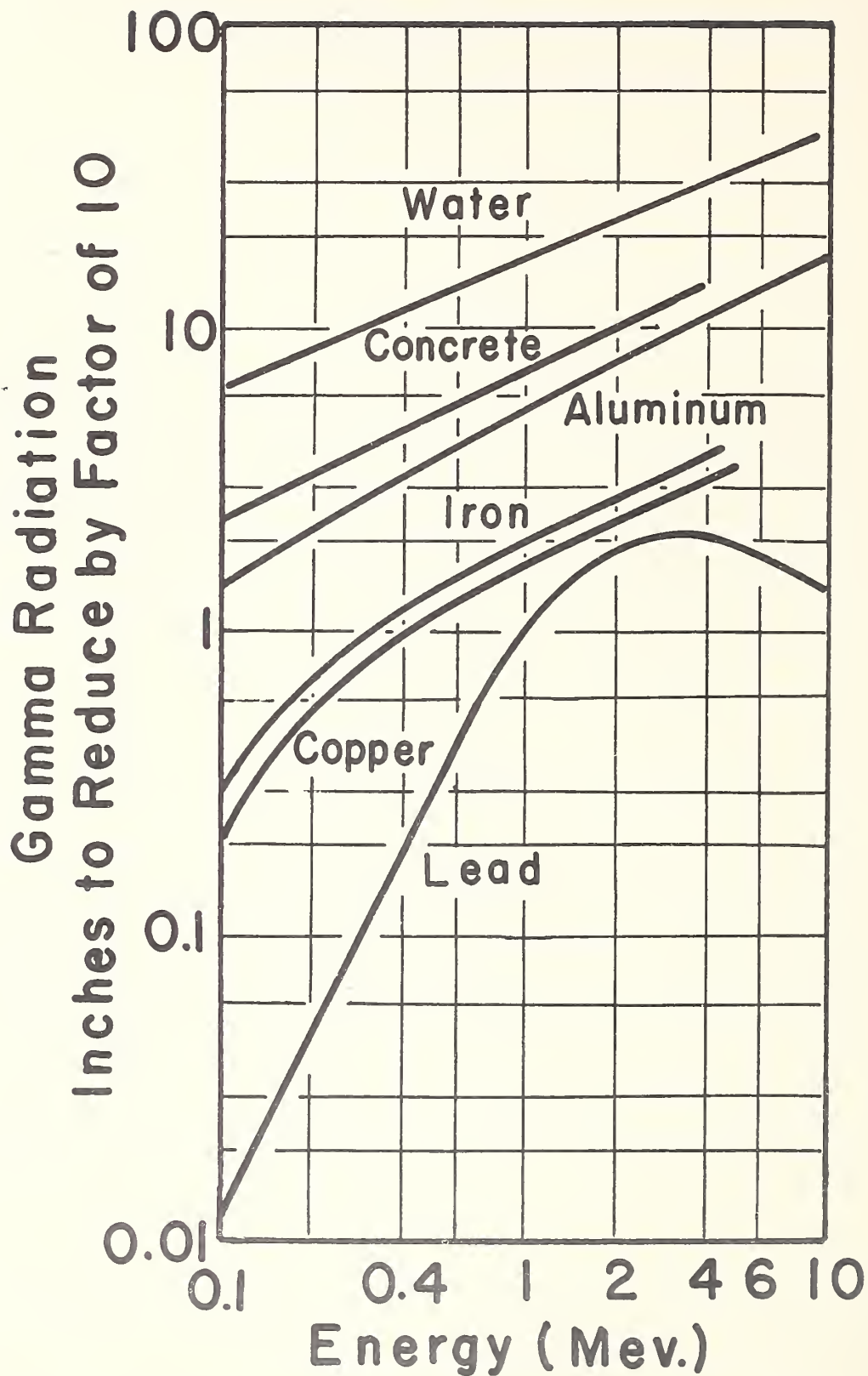


Figure 1.--The thickness of the indicated materials necessary to reduce the gamma radiation by a factor of 10 plotted as a function of photon energy. (From SRI Report No. 361, "The Industrial Uses of Radioactive Fission Products.")

where  $I$  is the intensity of the incident beam and is proportional to  $dI$  and  $dx$ . The proportionality constant,  $\mu_L$ , is known as the linear absorption coefficient.<sup>4</sup>

Solving the differential equation with boundary conditions  $I = I_0$  at  $x = 0$  yields:

$$I = I_0 e^{-\mu_L x} \quad (2)$$

The value of  $I$  decreases when  $\mu_L$  (density) and/or  $x$  (thickness) increases.

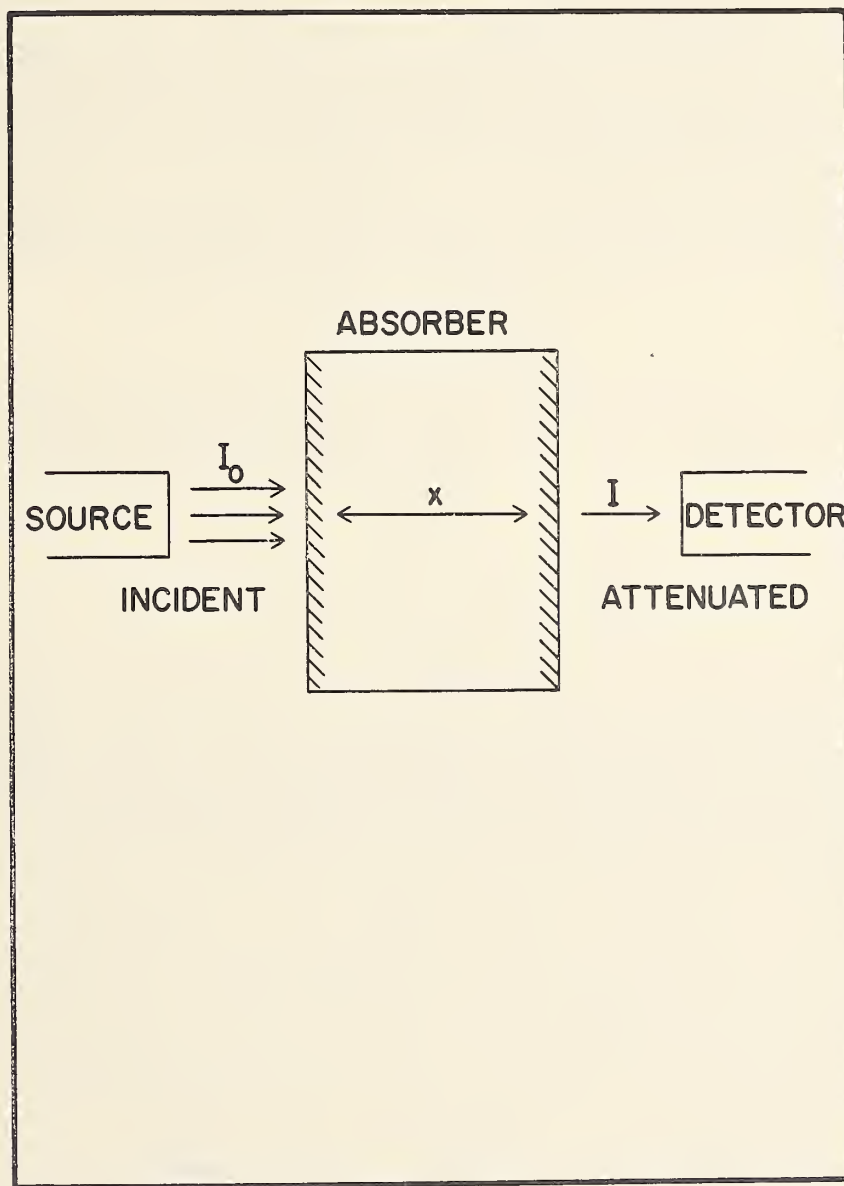


Figure 2.--Schematic drawing of radiation attenuation by an absorber.

<sup>4</sup>The absorption coefficient of a material is the product,  $N\sigma$ , of the number of atoms ( $N$ ) per unit volume and the cross section ( $\sigma$ ). The mass absorption coefficient is represented by  $\frac{N\sigma}{\rho}$  where  $\rho$  is the density of the absorber. If more than one process is involved,  $\sigma$  is replaced by the sum of the component cross sections or  $\sigma_{\text{tot}}$ . The linear absorption coefficient,  $\mu_L$ , is  $N\sigma_{\text{tot}}$ .



The change in I for a small change in mass per unit volume may be defined as sensitivity, S. The value of S can be expressed:

$$S = \Delta I / I_0 = (\mu + \Delta\mu) \times -e^{-\mu x} \quad (3)$$

Hence, for a given photon energy and a given absorber, the sensitivity is a constant fraction of the incident beam  $I_0$  and can be increased by increasing the source size. The upper limit of stable detector performance and the physical size of the necessary shielding restrict the source size.

At times it is desirable to operate radiation detection equipment near maximum sensitivity. However, the actual precision of any recording instrument is less dependent on the magnitude of the change in signal than on the ratio of the change to the constant fluctuation inherent in the system.

The concepts of half thickness,  $x_{\frac{1}{2}}$ , and mean range,  $\bar{R}$ , are useful in studies of gamma ray absorption. The half thickness,  $x_{\frac{1}{2}}$ , is that thickness of the absorber required to absorb half of the incident photons. It is expressed as:

$$x_{\frac{1}{2}} = \frac{\ln 2}{\mu} = \frac{0.693}{\mu} \quad (4)$$

The mean range,  $\bar{R}$ , is the average distance traveled by a photon before it is absorbed.  $\bar{R}$  may be shown to equal:

$$\bar{R} = \frac{1}{\mu} \quad (5)$$

This may also be written:

$$\bar{x} = \frac{1}{\mu} \quad (6)$$

Disintegrations of a radioactive material do not occur at evenly spaced time intervals, but randomly. The number of disintegrations in any particular time unit may vary widely from the number in any other similar time unit. Since the occurrence of events in radioactivity is random, the probability of any sequence of events can be predicted by theory. The probability theory states that the error associated with a single determination is proportional to the square root of the total number of counts:

$$E = K \sqrt{N} \quad (7)$$

where N = total number of counts and K = confidence factor. The value assigned to K may be obtained from the following:

<u>Error name</u>	<u>K</u>	<u>Confidence level</u>
Probable error	0.6745	50%
Standard error	1.0000	68.3%
Reliable error	1.645	90%
-	1.960	95%
-	2.575	99%

The standard error (K = 1.00) is commonly used in evaluating precision of counting measurements. Using longer counting times will reduce the standard error. With a given level of activity, the time necessary to achieve a desired level of confidence in the counting rate can be calculated.



## Types of Systems

Basically, there are four types of gages available for DXT work (5). These, together with the type of detector commonly employed, are:

Beta (usually Sr-90)	Ionization chamber
Bremsstrahlung (commonly Sr-90 and Fe)	Ionization chamber or scintillation photomultiplier
X-ray (commonly 10-100 Kev.)	Scintillation photomultiplier
Gamma ray (Cs-137, Co-60, etc.)	Scintillation photomultiplier

Before the source size, collimation size, and time constants for a DXT system can be determined, the sensitivity and resolution requirements must be known. Generally speaking, maximum sensitivity, within the bounds of economy and personnel health consideration, is sought. Equation 3 can be solved and evaluated by maximizing  $x$  as  $\Delta\mu$  approaches zero as a limiting value, using a Taylor series expansion. Then:

$$X = \frac{1}{\mu} \text{ or } \mu X = 1 \quad (8)$$

For any specific source, the theoretical maximum sensitivity occurs when the product of  $\mu$  and  $X$  is unity. Generally the resolution time of the DXT system will be determined by the characteristics of the electronic components of the system. The resolution time is determined experimentally for each system.

## Design of a DXT System

Consider a density gage, DXT, for sediment research that is capable of distinguishing differences in density of the order of 0.01. This discrimination should apply in the range of 60 to 120 pounds per cubic foot. This differentiation is about 0.6 pound per cubic foot of water, or some 0.1 inch of sediment per foot depth of water where the density of the unconsolidated sediment is assumed as 1.0. These dimensions are considered to represent a feasible goal. To meet these requirements, the radiation source and the detection device selected must have certain characteristics.

Source characteristics.--Two gamma-emitting radioisotopes are commonly employed in DXT gages. They are Co-60 and Cs-137. One may refer to a standard chart of the nuclides (3, 10) and find that Co-60 has two gamma photons of 1.17 and 1.33 Mev. energy and that Cs-137 has one photon (actually Ba-137) of 0.66 Mev. energy. The decay scheme of Co-60 (10) indicates that, for all practical purposes, one 1.17 Mev. photon is formed for each 1.33 Mev. photon observed. Thus for each disintegration in the Co-60 nucleus, two photons are emitted. The number of disintegrations per unit time for Co-60 will be much greater than for an equal number of Cs-137 atoms because of this decay scheme. Furthermore, the number of disintegrations per unit time for any two radioisotopes is inversely proportional to the half-lives of the same radioisotopes. This advantage of a higher counting rate for Co-60 use is offset by the correction necessary for source strength with time due to its shorter half-life ( $t_{\frac{1}{2}}$  Co-60 = 5.2 years,  $t_{\frac{1}{2}}$  Cs-137 = 30 years).

Both Co-60 and Cs-137 sources are readily available through commercial channels. The costs of such sources are comparable, about \$100 each for a 5 or 10 millicurie source. Co-60 sources of greater radioactivity cost somewhat more than do similar Cs-137 sources. The advantages inherent to either Co-60 or Cs-137 do not appear great enough to distinctly favor one isotope over the other.

Attenuation.--Van Bavel and others (12) have discussed the problem of detecting monoenergetic primary radiation. In order to effectively study the attenuation of the primary radiation, the detection system must be capable of discrimination. Only nonabsorbed, nonscattered radiation is to be recorded. The DXT system design must then be capable of distinguishing wavelengths and have the ability to eliminate those not wanted. This can be accomplished by using a scintillation detector and a pulse height analyzer.

The beam of gamma rays from the source must be focused, or collimated, so that a rather narrow beam of rays is passed through the absorber medium to the detector. This collimating can be accomplished by placing about the source sufficient lead in a suitable arrangement. Only those photons emitted in the direction of the desired beam can escape from the confines of the lead shielding.

Absorber characteristics.--Gamma rays are attenuated, absorbed, in passing through water. It is possible to predict the absorption that will occur (2). The number of Co-60 1.33 Mev. photons passing through a 4-foot column of water is about 0.01 that of the incident intensity; for Co-60 1.17 Mev. photons the percentage would be somewhat less. For 0.66 Mev. Cs-137 photons, only about 0.001 of the incident photons will emerge.

The sediment within the water column will also absorb photons. The number of such photons absorbed will be directly proportional to the density of sediment present. The absorption of photons is also dependent on the atomic number of the atoms present. In sediment generally only atoms of low atomic number (less than 25) are found in any abundance. Hence, the absorption of photons in measurements of sediment density can be directly correlated with the mass of sediment.

Detection equipment.--The detector system must be capable of distinguishing monochromatic radiation. The photon beam must be collimated, and the depth of the absorption medium or geometry of the system must remain constant.

Consider a 10-millicurie source of Cs-137. Approximately  $3.7 \times 10^8$  photons are emitted each second from this source. It was indicated that in passing through 4 feet of water only 0.001 of the incident photons would be detected on emergence, i.e.,  $3.7 \times 10^5$  photons per second. The geometry or efficiency of a Geiger-Mueller system will seldom exceed 1 percent for gamma rays. Scintillation methods may be designed to yield greater efficiencies. The photons "seen" by the detector and counted, considering the lowest probable efficiency, would approximate  $3.7 \times 10^3$ /second, or some 200,000 c.p.m. This value is sufficiently large that a 1-percent attenuation due to sediment concentration would be detected by the counting circuit.

The calculated performance of the planned system is based on conservative estimates of the parameters involved. A system designed to be one-tenth as sensitive might also perform satisfactorily. However, the estimates of sensitivity employed are not believed to be overrated or unrealistic.

## LABORATORY STUDIES

Densities of soil have been measured by two general gamma-attenuation methods. One is the scattering procedure, which may be applied to surface as well as depth measurements (8). The other is the transmission method (11, 12), which is largely the DXT system adapted to the problem of measuring soil density. The attenuation of the gamma radiation in either case is governed by Equation 2. The value of  $\mu_L$  is readily determined for each method--the value of  $x$  is not readily determined for the scattering

procedure. An empirical relationship between density and radiation attenuation is obtained in the scattering procedure. A calibration curve is then prepared. This calibration holds only so far as the geometry of the system and the composition of the materials tested remain unchanged.

Instruments employing the gamma radiation scattering method for determining density are in use at the USDA Sedimentation Laboratory. One procedure employs the Model P-20 depth density probe manufactured by the Nuclear-Chicago Company.<sup>5</sup> This probe is designed to measure soil density. The use of an access tube is recommended. A second operation uses an instrument manufactured by Technical Operations, Inc.,<sup>5</sup> for measuring densities of sediments under water. Both instruments are used in connection with the Model 2800 portable scaler manufactured by Nuclear-Chicago. The calibration and use of the P-20 depth density probe in Mississippi has been reported.<sup>6</sup> A description and an evaluation of the Technical Operations, Inc., instrument, made to Beach Erosion Board specifications and subsequently modified for use, has been reported also by this Laboratory (4) and by the Beach Erosion Board (1).

The sediment density probe, of Beach Erosion Board design, has been found to yield excellent results (4). However, the design (length) of the probe precludes density measurements of any layers less than about 16 inches in depth. The need for a DXT instrument for sediment research that can be used to evaluate densities of much thinner layers is apparent. The transmission method has shown promise for this purpose.

Two systems have been used in this Laboratory for determining densities by the direct transmission method. One of these systems is essentially that described by Van Bavel (11) and Van Bavel and others (12). The other system utilizes a scintillation probe containing a NaI (Tl) crystal mounted on a 6291 DuMont photomultiplier tube and the counting circuitry of a Baird-Atomic differential spectrometer. The former system has been transistorized for field use. The latter system is extremely versatile, and the operator has facilities for making precise measurements of any monoenergetic gamma radiation desired.

Van Bavel (11) and Van Bavel and others (12) have shown that not only is the two-probe, or transmission, method for measuring soil densities highly reliable, but its performance is predicted by Equation 2:

$$I = I_0 e^{-\mu_L x} \quad (2)$$

Measurements of sediment density were made in the USDA Sedimentation Laboratory utilizing the transmission technique. The source employed was 10 mc. of Cs-137 with the photon beam collimated by lead bricks. The collimated beam passed vertically through a 55-gallon steel drum and was detected with a scintillation probe (1 $\frac{1}{4}$ -inch diameter x 1-inch thick NaI (Tl) crystal) and was recorded by a Baird-Atomic single channel scintillation spectrometer. The measurement of the 0.66 Mev. Cs-137 emission was made with the spectrometer operating differentially. Generally, the gate width was 2 volts. The measurement of the Cs-137 radiation thus was confined to a small portion of the peak of the emission spectrum. Little, if any, scattered radiation was measured.

The attenuation of Cs-137 emission by water can be estimated from Figure 1. For 0.662 Mev. photons, some 14 inches are calculated to attenuate the transmitted gamma ray tenfold. In the experiment setup, 12 inches of water reduced the gamma

<sup>5</sup> Reference to commercial equipment or the manufacturer is in no way an endorsement by the U. S. Department of Agriculture.

<sup>6</sup> McHenry, J. R. 1960. Calibration and use of a gamma probe for soil density measurements. Agr. Res. Serv. Res. Rpt. No. 341.



transmission elevenfold. The Cs-137 emission had passed first through the steel drum, so that the attenuation measured is not strictly that of a Cs-137 beam in water only.

The attenuation of gamma rays as a function of sediment density is shown in Figure 3. Six inches of water plus the indicated percentage by weight of a Grenada soil were placed in the steel drum. The ratio,  $I_w/I_s$ , was plotted against percentage of added sediment.  $I_w$  is the intensity of the gamma ray beam passing through the 6 inches of water;  $I_s$  is the intensity of the gamma ray beam passing through the 6 inches of water containing the given percentage of sediment. The plot of  $I_w/I_s$  versus sediment percentage follows the exponential relation  $I = I_0 e^{-\mu x}$  as predicted.<sup>7</sup>

Similarly, the data in Figure 4 show the exponential relationship to hold for sediment depths of 12 inches. The slope of the curve for attenuation in 12 inches of water is twice that for 6 inches of water, as predicted by theory. The data plotted in Figure 4 are based on additions of sand as sediment. Because of the physical difficulty in obtaining a uniform depth of sand, the results are somewhat erratic, particularly at the lower sediment percentages. Sand was used rather than soil as the sediment material to illustrate that attenuation by sediment was independent of size of material so long as the chemical composition, i.e., absorption coefficient, was similar.

The data indicate excellent agreement between experimental results and theoretical values when monoenergetic gamma rays are measured. The differential spectrometer provides an excellent means for such measurements.

## FIELD INSTALLATIONS

The laboratory tests indicated that field determinations of sediment densities are feasible. Furthermore, these determinations can be made on successive vertical layers of the order of an inch or two in depth by use of the direct transmission, or two-probe, system. Electronic equipment is available at the USDA Sedimentation Laboratory that permits the measurement of monoenergetic gamma emissions only so the attenuation can be expressed by Equation 2.

A two-probe system similar to that described by Van Bavel (11) and Van Bavel and others (12) is also available. Measurements of sediment densities in plot runoff silt boxes have been made using this equipment. Densities of successive vertical layers of as little as 1 inch in thickness were obtained with an accuracy better than 1 percent. The attenuated and discriminated gamma emission measured was a function only of the total density of the material between the source and the detector. The attenuation due to water was taken as constant--any increased attenuation was, therefore, a function of the increased density due to the presence of sediments. The calibration curve was constructed from attenuation data obtained in media of known density.

Assuming the total absorption coefficients of the sediments to be similar, because of similar chemical composition, density values were obtained directly from the calibration curve upon measurement of the attenuated gamma emissions. Similar specific gravities are also assumed for the sediments used in constructing the calibration curve and for those whose densities are to be determined. The results of these field determinations of sediment densities by the two-probe method of gamma-ray attenuation will be published in a later report.

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<sup>7</sup> A theoretical evaluation of the experimental results is presented in the Appendix at end of this publication.

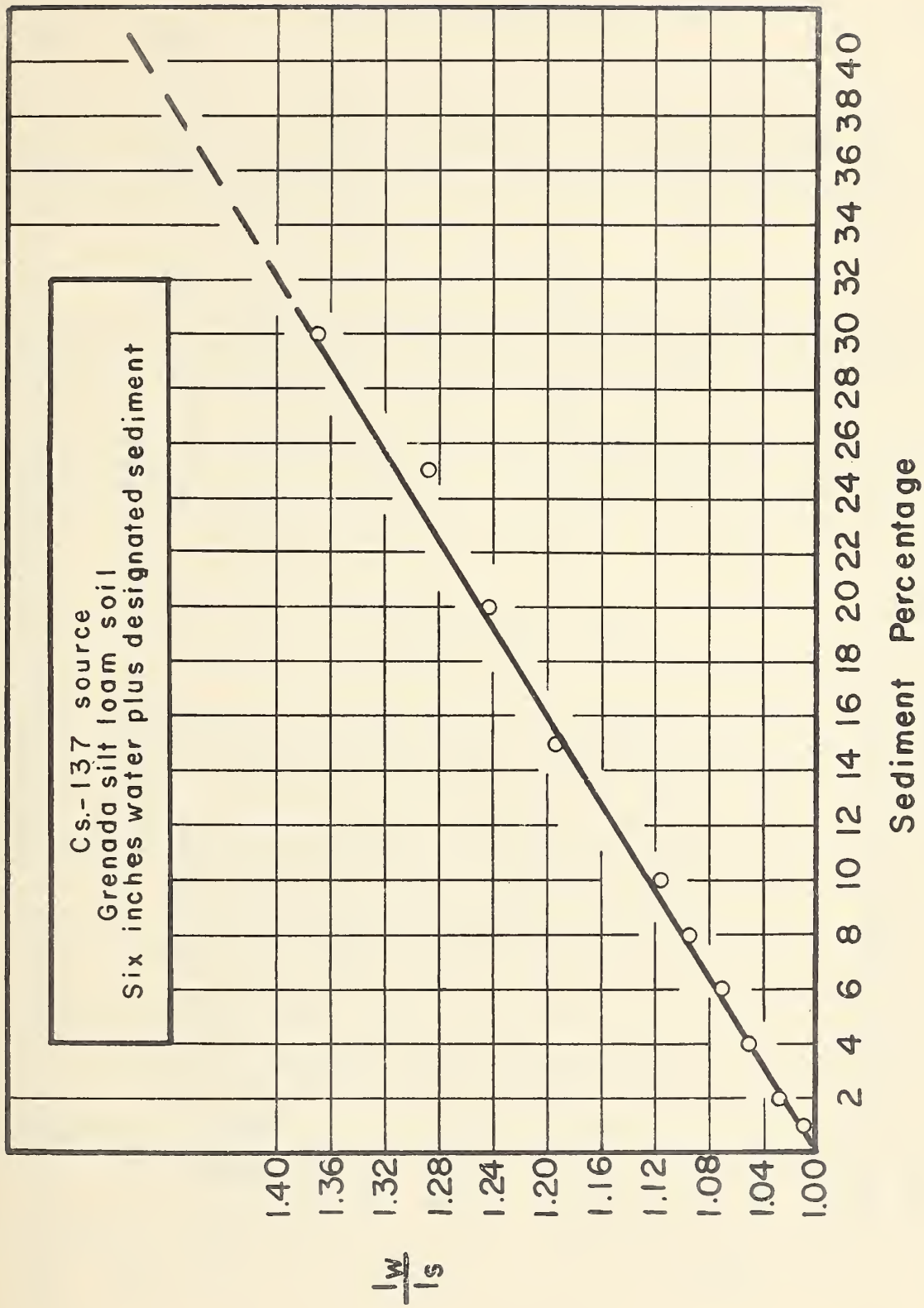


Figure 3.--The measured gamma activity density, as the ratio of the intensity in water,  $I_w$ , to the intensity of sediment in water,  $I_s$ , plotted as a function of the sediment concentration.

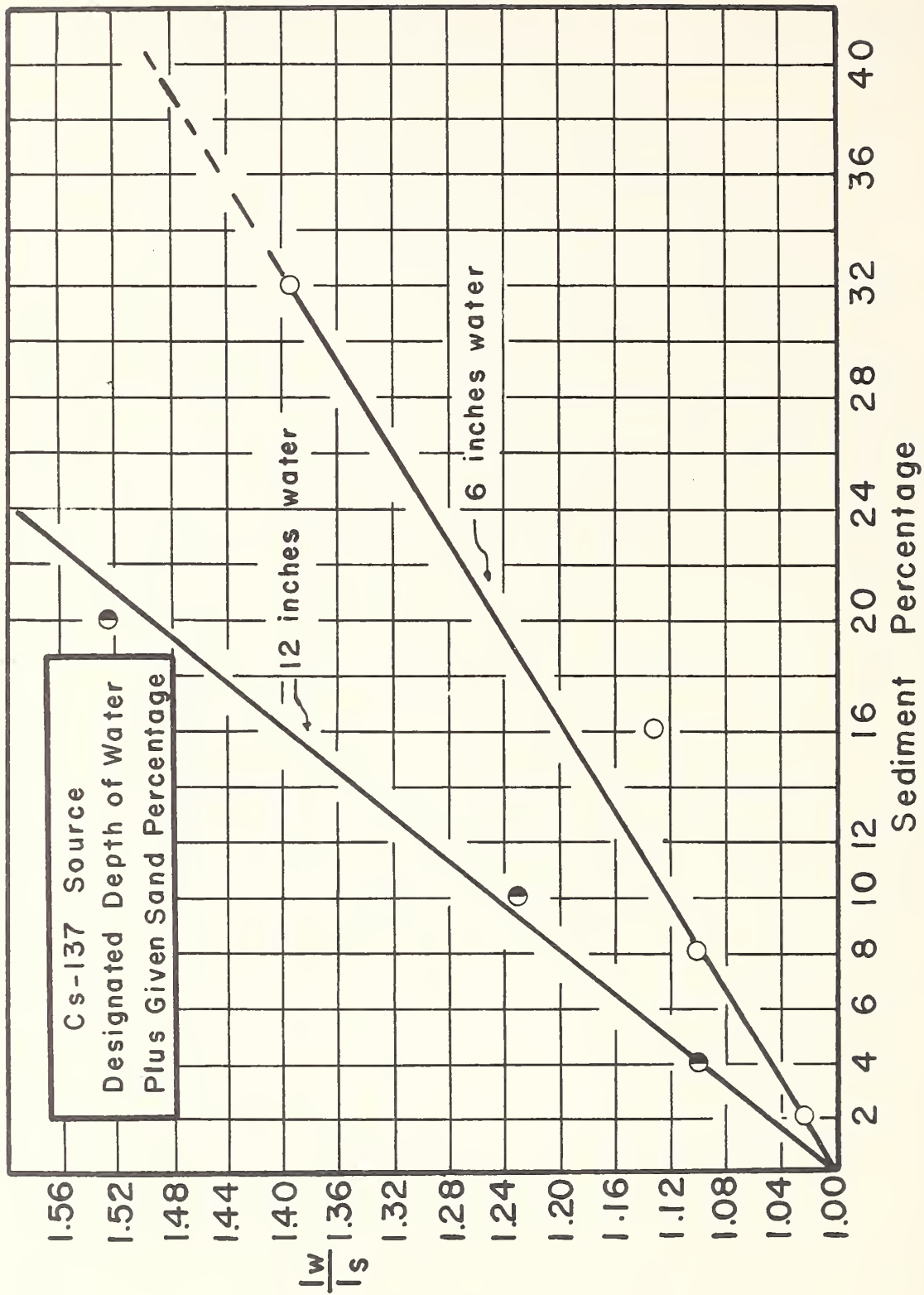


Figure 4.--The measured gamma activity density, as the ratio of the intensity in water,  $I_w$ , to the intensity of sediment (Pigeon Roost Creek sand) in water,  $I_s$ , plotted as a function of the sediment concentration.



## SUMMARY AND CONCLUSIONS

Radiation gages employing radioactive material have been developed for the measurement of density or thickness of various materials. The possibility of utilizing such a system for evaluating the sediment density or concentration in silt boxes, laboratory flumes, and flowing streams has been investigated. The attenuation of gamma rays follows the exponential equation,

$$I = I_0 e^{-\mu_L x} \quad (2)$$

as long as only the primary emissions are considered. To effect this type of measurement, a pulse height analyzer or discriminator is necessary to reject unwanted secondary radiations. Systems employing these electronic features were successful in laboratory operation in measuring sediment density or concentration.

A two-probe system employing these electronic features has been used in field studies of the sediment densities in silt boxes. Equipment sufficiently rugged for extended field use will be required to implement these preliminary studies.

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## APPENDIX

The intensity of the radioactive beam after passing through water is:

$$I_w = I_o e^{-\mu_w x_w} \quad (9)$$

where  $I_o$  = incident intensity,  $\mu_w$  = linear absorption coefficient for water,  $x_w$  = thickness of the water. If  $\mu_w$  is taken as equal to  $k \rho_w$  where  $k$  is a constant and  $\rho_w$  is the density of water, the expression becomes:

$$I_w = I_o e^{-k \rho_w x_w} \quad (10)$$

Assume  $P$  is the ratio of the soil weight to the water weight times one hundred, that is,  $P = [\text{soil wt.}/\text{water wt.}] \times 100$ , and the intensity of the radioactive beam after passing through soil and water is:

$$I_{w+s} = I_w e^{-k_s \rho_s x_s} = I_o e^{-(k_w \rho_w x_w + k_s \rho_s x_s)} \quad (11)$$

where  $\rho_s$  = the density of soil and  $x_s$  = the thickness of the soil. As  $\rho_s x_s = \rho_w x_w P/100$ , the expression can be stated:

$$I_{w+s} = I_o e^{-k_w \rho_w x_w (1 + \frac{k_s \cdot P}{k_w \cdot 100})} \quad (12)$$

Differentiate with respect to  $P$ :

$$\frac{d[I_{w+s}]}{dP} = \frac{-k_s \rho_w x_w}{100} \cdot I_{w+s} \quad (13)$$

or

$$dP = \frac{-100}{k \rho_w x_w} \cdot \frac{dI_{w+s}}{I_{w+s}} \quad (14)$$

Equation 11 may be expressed in log form as:

$$\ln \left[ \frac{I_{w+s}}{I_w} \right] = -k_s \rho_s x_s = \frac{-k_s \rho_w x_w P}{100} \quad (15)$$

or

$$\ln \left[ \frac{I_w}{I_{w+s}} \right] = k_s \rho_w x_w \frac{P}{100} \quad (16)$$

and this becomes:

$$\log_{10} \left[ \frac{I_w}{I_{w+s}} \right] = 0.434 k_s \rho_w x_w \frac{P}{100} \quad (17)$$

and finally:

$$P = \frac{230}{k_s \rho_w x_w} \log_{10} \left[ \frac{I_w}{I_{w+s}} \right] \quad (18)$$

If the data in Figures 3 and 4 are plotted on semilog paper in accordance with Equation 17 where  $\log_{10} \left[ \frac{I_w}{I_w + s} \right]$  is the ordinate and  $x_w P$  is the abscissa, the slope of the plot will be found to be:

$$\text{slope} = \frac{1}{110} = \frac{0.434 k_s \rho_w}{100}$$

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Now if  $k_s$  is evaluated, using the slug as the unit of mass and the value for  $\rho_w = 1.94$  slugs/ft.<sup>3</sup>, then:

$$k_s = 1.07 \text{ ft.}^2/\text{slug}$$

Substituting the experimentally determined value of  $k_s$  into Equation 14, the expression becomes:

$$dP = \frac{-48}{x_w} \cdot \frac{d \left[ \frac{I_w + s}{I_w + s} \right]}{I_w + s} \quad (19)$$

with  $x_w$  being expressed in feet. In terms of parts per million, the equation becomes:

$$d \text{ (p.p.m.)} = \frac{-5 \times 10^5}{x_w} \cdot \frac{d \left[ \frac{I_w + s}{I_w + s} \right]}{I_w + s} \quad (20)$$

For a 1-foot thickness of water,  $x_w = 1$ , to measure to 500 p.p.m. (assumed concentration in streams desired to measure), the change in radiation intensity,  $\frac{d \left[ \frac{I_w + s}{I_w + s} \right]}{I_w + s}$ , would have to be measured to:

$$\frac{5 \times 10^2}{5 \times 10^5} \times 100 = 0.1\%$$

In order for the standard error of a single measurement not to exceed this value, a total count in excess of one million ( $N = 10^6$ ) is necessary.

The above calculations are based on the experimental data obtained with a small ( $1\frac{1}{2} \times 1$ -inch) NaI crystal in conjunction with a single channel scintillation spectrometer operating differentially. Increased count rates for the system could be obtained by:

- Using a larger crystal, which would yield higher sensitivity.
- Employing a larger (more powerful) radioactive source.
- Operating the scintillation spectrometer integrally, using the base line discriminator only to cut out energies below that of the cesium-137 peak.

The point of interest here is that the device employed is not the ultimate in sensitivity or precision. The results do indicate, however, the limitations of the instrument employed; and, as indicated, the lines of probable fruitful research are enumerated.



